

An Effective Theory for the Four-Body System

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Abstract. We consider the non-relativistic four-body system with large scattering length and short-range interactions within an effective theory with contact interactions only. We compute the binding energies of the ^4He tetramer and of α -particle. The well-known linear correlation between the three-body binding energies and the four-body binding energies of these physical systems can be understood as a consequence of the absence of a four-body force at leading order.

Introduction - Effective theories are particularly well suited to describe low-energy properties of physical systems in a model-independent way. Results and errors can be improved systematically and thus, effective theories can be used in principle to compute observables to arbitrary high precision. If the scattering length a of two particles is much larger than the typical low-energy length scale ℓ of the system, one can use an effective theory with contact interactions only, to compute observables in an expansion in ℓ/a . The beautiful feature of this theory is, that it doesn't make any assumptions about the underlying physics, besides that the resulting potential is short-ranged and produces a large scattering length. This allows for a systematic comparison of low-energy systems at different length scales. Particularly interesting, in this respect, are few-body systems with large scattering length: For the three-body system it turns out that a further piece of three-body information is needed to fully describe observables. In the effective theory this is reflected by a leading order three-body interaction which has to be renormalized accordingly. Its renormalization group flow is governed by a limit cycle. In [1] we considered the four-boson system within this framework. Here we will summarize the most important results without going into the technical details and will also present results from the four-nucleon sector as recently given in [2].

Four-Boson System - At leading order the regulated effective low-energy potential generated by a non-relativistic effective field theory with short-range interactions is given by

$$\langle \mathbf{u}_1 | V | \mathbf{u}'_1 \rangle = \langle \mathbf{u}_1 | g \rangle \lambda_2 \langle g | \mathbf{u}'_1 \rangle, \quad (1)$$

where \mathbf{u}_1 and \mathbf{u}'_1 are the relative three-momenta of the incoming and outgoing particles, respectively. $g(\mathbf{u}_1) = \exp(-u_1^2/\Lambda^2)$ is a regulator function which suppresses momentum contributions with $u_1 \gg \Lambda$. The coupling constant λ_2 can be matched to the scattering length or the two-body binding energy. The three-body force (which is needed to renormalize the three-body system) is given by $V_3 = |\xi\rangle \lambda_3 \langle \xi|$, where $\xi(\mathbf{u}_1, \mathbf{u}_2) = \exp(-(u_1^2 + \frac{3}{4}u_2^2)/\Lambda^2)$ is the corresponding regulator function and \mathbf{u}_2 repre-

TABLE 1. Binding energies of the ^4He trimer and tetramer in mK. The two right columns show the results by Blume and Greene [3] (denoted by the index BG) while the two left columns show our results. The number in brackets was used as input to fix λ_3 .

system	$B^{(0)}$ [mK]	$B^{(1)}$ [mK]	$B_{\text{BG}}^{(0)}$ [mK]	$B_{\text{BG}}^{(1)}$ [mK]
$^4\text{He}_3$	127	[2.186]	125.5	2.186
$^4\text{He}_4$	492	128	559.7	132.7

sents the second Jacobi momentum in the three-body system. The three-body coupling constant λ_3 can be fixed by demanding that the binding energy of the shallowest three-body bound state stays constant as the cutoff Λ is changed. This renormalization prescription leads to the limit cycle mentioned above.

We have used the Yakubovsky equations to compute the binding energies of the $^4\text{He}_4$ tetramer. As three-body input we used the energy of the shallowest three-body bound state as calculated by Blume and Greene (BG) [3]. An analysis of the cutoff dependence of the results for the binding energies shows that no four-body force is needed to renormalize the four-body sector. Further, a comparison of our results with the values obtained by BG [3] is shown in Table 1. The results of their calculation for the trimer and tetramer are given in the two right columns of Table 1, while our results are given in the two left columns. In general, our results are in good agreement with the values of BG. We also analyzed the correlation between three-body and four-body binding energies. The left plot in Fig. 1 shows the ground state energy $B_4^{(0)}$ of the ^4He tetramer as a

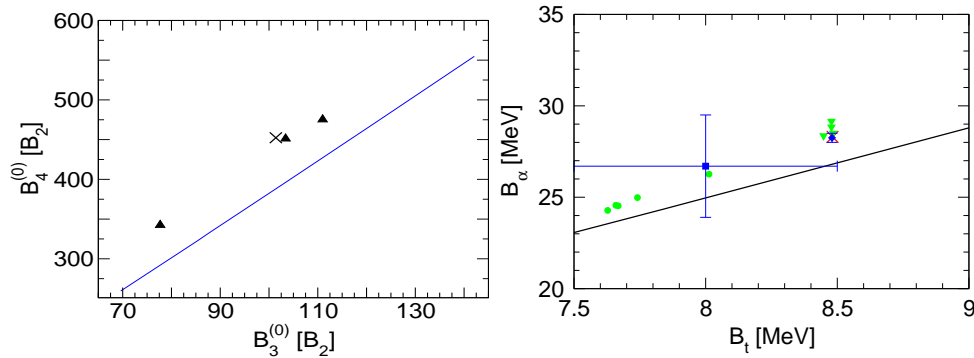


FIGURE 1. The left plot shows the correlation between the ground state energies of the ^4He trimer and tetramer. The solid line shows the leading order effective theory result and the cross denotes the calculation for the LM2M2 potential by Blume and Greene [3]. The triangles show the results for the TTY, HFD-B, and HFDHE2 potentials [4, 5]. The right plot shows the correlation between the triton and the α -particle binding energies. The solid line shows our leading order result using the singlet scattering length a_S and the deuteron binding energy B_d as two-body input. The grey dots and triangles show various calculations using phenomenological potentials without our including three-nucleon forces, respectively [6]. The squares show the results of chiral EFT at NLO and N²LO [7, 8] while the cross shows the experimental point.

function of the trimer ground state energy $B_3^{(0)}$. The solid line is the leading order result of our effective theory calculation and the cross denotes the result of the calculation by BG for the LM2M2 potential [3]. For the ground states of the trimer and tetramer, calculations with other ${}^4\text{He}$ potentials are available and are shown as well [4, 5].

Results for the Four-Nucleon System - It is straightforward to apply the above procedure to the four-nucleon system. Without going into the technical details of the inclusion of spin and isospin we present our results: For the α -particle we obtain a binding energy $B_\alpha = 26.9$ MeV if we use the triplet scattering length, the deuteron binding energy and triton binding energy as input. By varying the three-body binding energy we are again able to observe a linear correlation between the three-body and four-body binding energies, which is shown in the right plot of Fig. 1. Thus, we conclude that the Tjon line is a result of the large scattering lengths in the nucleon-nucleon system. More information and further references on this topic can be found in [2].

Summary - We have shown, that four-body systems with large scattering length can be described within the framework of the effective theory with contact interactions. Our results for the binding energies of the ${}^4\text{He}$ tetramer and the α -particle are in good agreement with theoretical calculations and the experimental value, respectively. Universal properties of these systems like the Tjon line turn out to be a result of the large scattering length in the two-body sector. This can be understood in detail by the absence of a four-body force at leading order. As a consequence, all two-body interactions that produce a large scattering length in the two-body sector will give four-body binding energies lying close to the Tjon line.

In the near future further effort should be devoted to the computation of scattering observables and the inclusion of effective range corrections.

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